MATH 105 – SECOND EXAM SOLUTIONS

February 26, 2009

NAME: ______________________________

INSTRUCTIONS:

(1) SHOW ALL WORK.
(2) Do not begin until instructed to do so.
(3) You have 80 minutes to complete the exam.
(4) You may use a calculator unless otherwise indicated.
(5) When an **exact** answer is specified, a calculator approximation is not acceptable.

PROBLEM	POINTS	SCORE
1	8	
2	5	
3	8	
4	10	
5	10	
6	10	
7	13	
8	16	
9	20	
EC	10	
TOTAL	100	

1. (8 points) Identify the following as an example of deductive or inductive reasoning.

(a) Since November, December, and January have been snowier than average this season, February and March will also be snowier than average.

Since we are looking at specific events and making a general conclusion, the above is an example of **inductive** reasoning.

(b) The Fibonacci sequence is a list of numbers starting with 1, 1, 2, 3, 5, 8, . . . The definition of the sequence says that the next number is the sum of the previous two numbers. Therefore, 13 follows 8.

Since we are looking at the general definition and making a specific conclusion, the above is an example of **deductive** reasoning.

2. (5 points) Use an English sentence to write the negation of the following statement using an existential quantifier.

"All bears eat honey."

"Some bears do not eat honey."
OR
"There exists a bear who does not eat honey."

3. (8 points) Write the converse and the contrapositive of the following statement. Be sure to label which is which.

"If pigs fly, then hippos rule the world."

CONVERSE: "If hippos rule the world, then pigs fly."

CONTRAPOSITIVE: "If hippos do not rule the world, then pigs do not fly."

4. (10 points) Use the statements p, q, and r to write the logical symbols below as English sentences.

p: Today is Thursday.
q: It is the afternoon.
r: Next week is Spring Break.

(a) $(\sim r) \wedge q$

"Next week is not Spring Break , and it is the afternoon."

(b) $p \leftrightarrow (r \vee \sim q)$

"Today is Thursday if and only if next week is Spring Break or it is not the afternoon."

5. (10 points) Write a full truth table for: $\sim (p \to q) \vee p$.

p	q	$p \to q$	$\sim (p \to q)$	$\sim (p \to q) \vee p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F

6. (10 points) Assuming that p is true, q is false, and r is false, decide whether or not the following is true or false:

$$(q \leftrightarrow \sim p) \to (\sim r \vee q)$$

p	q	r	$\sim p$	$q \leftrightarrow \sim p$	$\sim r$	$\sim r \vee q$	$(q \leftrightarrow \sim p) \to (\sim r \vee q)$
T	F	F	F	T	T	T	T

Based on the last column of the table, the statement is **TRUE**.

7. (13 points) Use a truth table to verify whether or not the following argument is valid.

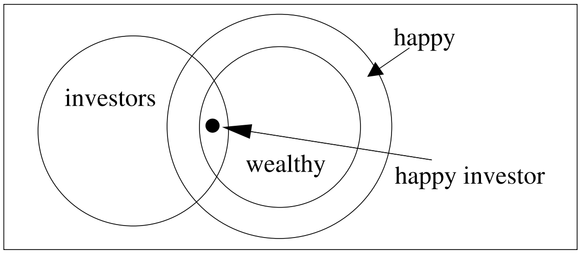
$$\begin{array}{l} p \vee q \\ \sim p \to \sim q \\ \hline \therefore p \end{array}$$

p	q	$p \vee q$	$\sim p$	$\sim q$	$\sim p \to \sim q$	$(p \vee q) \wedge (\sim p \to \sim q)$	$[(p \vee q) \wedge (\sim p \to \sim q)] \to p$
T	T	T	F	F	T	T	T
T	F	T	F	T	T	T	T
F	T	T	T	F	F	F	T
F	F	F	T	T	T	F	T

Since every entry in the last column of the truth table is **TRUE**, the argument is valid.

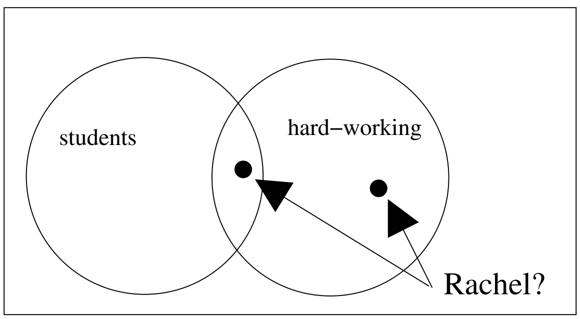
8. (16 points) Use an Euler diagram to check the validity of each syllogism.

(a) Some investors are wealthy.
All wealthy people are happy.
∴ Some investors are happy.



Since the overlap of investors and wealthy falls inside of the happy group, there will be happy investors and the argument is valid.

(b) Some students are hard-working.
Rachel is hard-working.
∴ Rachel is a student.



The diagram allows us to place Rachel in two different places, one that is inside the student group and one that is not. Therefore, the argument is invalid.

9. (20 points) Use either truth tables or DeMorgan's Laws to show whether or not each pair of statements is logically equivalent.

(a) $\sim (p \wedge \sim q)$ and $(\sim p) \vee q$

By DeMorgan's Law, $\sim (p \wedge \sim q)$ is logically equivalent to $(\sim p) \vee \sim (\sim q)$, which is logically equivalent to $(\sim p) \vee q$. One may also use a truth table to verify the equivalence:

p	q	$\sim q$	$p \wedge \sim q$	$\sim (p \wedge \sim q)$	$\sim p$	$(\sim p) \vee q$
T	T	F	F	T	F	T
T	F	T	T	F	F	F
F	T	F	F	T	T	T
F	F	T	F	T	T	T

As columns 5 and 7 are identical, the statements are logically equivalent.

(b) $p \rightarrow \sim q$ and $(\sim p) \wedge (\sim q)$

p	q	$\sim q$	$p \rightarrow \sim q$	$\sim p$	$(\sim p) \wedge (\sim q)$
T	T	F	F	F	F
T	F	T	T	F	F
F	T	F	T	T	F
F	F	T	T	T	T

As columns 4 and 6 are different, the statements are not logically equivalent.

EC. (10 points) Complete the following syllogism from Lewis Carroll and use logical methods to demonstrate the validity. Start by listing basic statements symbolically and rewriting all sentences as implications. *Hint:* The syllogism contains 6 basic statements all about kittens.

No kitten, that loves fish, is unteachable.
No kitten without a tail will play with a gorilla.
Kittens with whiskers always love fish.
No teachable kitten has green eyes.
No kittens have tails unless they have whiskers.

$\therefore$ **If this kitten will play with a gorilla, then it does not have green eyes.**
or **Only a kitten without green eyes will play with a gorilla.**

Use the following variables:
p: This kitten loves fish.
q: This kitten is teachable.
r: This kitten has a tail.
s: This kitten will play with a gorilla.
t: This kitten has whiskers.
u: This kitten has green eyes.

Symbolically, the argument can be written as:

$p \to q$
$s \to r$
$t \to p$
$q \to \sim u$
$r \to t$

$\therefore s \to r \to t \to p \to q \to \sim u$
or $s \to \sim u$