

MATH 105 – FIRST EXAM SOLUTIONS
February 3, 2009

NAME: _____

INSTRUCTIONS:

- (1) SHOW ALL WORK.
- (2) Do not begin until instructed to do so.
- (3) You have 80 minutes to complete the exam.
- (4) You may use a calculator unless otherwise indicated.
- (5) When an **exact** answer is specified, a calculator approximation is not acceptable.

PROBLEM	POINTS	SCORE
1	4	
2	5	
3	15	
4	20	
5	6	
6	24	
7	6	
8	4	
9	4	
10	12	
TOTAL	100	

1. **(4 points)** Find a counterexample to the following statement to show that it is false.
 “If x and y are integers and $x < y$, then $x^2 < y^2$.”

One example is $x = -3$ and $y = -2$ as $x^2 = 9 > 4 = y^2$.

2. **(5 points)** Give an algebraic formula for each operation and then decide whether the two operations are the same or not. If they are different, give a counterexample.

- (i) Cubing twice the value of s .
 (ii) Doubling the cube of s .

The first statement is $(2s)^3$, and the second is $2s^3$. These are not the same as can be seen for $s = 1$, where the first gives 8 and the second gives 2.

3. **(15 points)** Use compatible numbers or rounding to estimate answers to the following questions. Start each part by writing the formula for the exact value. Using a calculator to find the exact answer is not acceptable.

(a) Amber fills her car’s gas tank with 9.35 gallons after driving 187 miles since the last fill-up. What is her car’s fuel efficiency in miles per gallon?

$$\frac{187}{9.35} \approx \frac{180}{9} = 20 \text{ mpg}$$

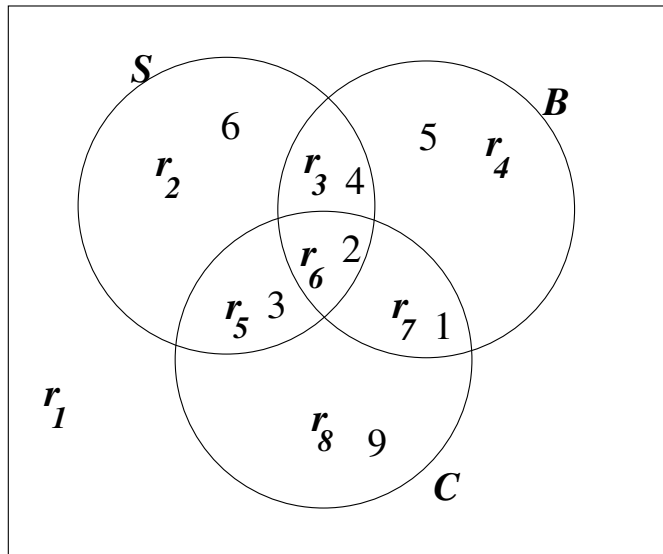
(b) Joseph takes his friends out to dinner and owes a bill of \$147.36. If he wants to tip at 15%, how much should he leave?

$$(\$147.36)(0.15) \approx \$150(0.15) = \$15 + \$7.50 = \$22.50.$$

(c) $(15.5 \times 18.5 + 15.5 \times 11)/140$

$$(15.5 \times 18.5 + 15.5 \times 11)/140 = (15.5 \times 29.5)/140 \approx (15 \times 30)/140 \approx 450/150 \approx 3$$

4. (20 points) A survey of 100 random people showed the following:
- 2 own stocks, bonds, and CDs
 - 6 own stocks and bonds
 - 5 own stocks and CDs
 - 3 own bonds and CDs
 - 15 own stocks
 - 12 own bonds
 - 15 own CDs
- Use the Venn diagram below to help answer the following questions.



- (a) How many people own only CDs?

Using the information above, one can fill in sizes for regions r_2 through r_8 . Then the number who own only CDs is the size of r_8 , which is 9.

- (b) How many people had no investments?

Since 100 people were surveyed and since regions r_2 through r_8 contain 30 people, region r_1 (the people with no investments) contains 70 people.

- (c) Use set notation to describe the set of all people who own stocks or bonds but no CDs. (Use S , B , and C , but not the regions r_i .)

$$(S \cup B) - C.$$

- (d) How many people own stocks or bonds but no CDs?

The set $(S \cup B) - C$ is made of regions r_2 , r_3 , and r_4 . Thus, 15 people fall in this category.

5. (6 points) Rewrite each set as indicated.

(a) Rewrite $\{n : n \text{ is an even whole number less than } 10\}$ using an element list.

$$\{0, 2, 4, 6, 8\}$$

(b) Rewrite $\{a, e, i, o, u\}$ using set-builder notation based on a common trait of the elements.

$$\{v : v \text{ is a vowel}\}$$

6. (24 points) Correct each false statement below to make it true.

(a) $5 \notin \{y : y \text{ is a real number}\}$

$$5 \in \{y : y \text{ is a real number}\}$$

(b) $\{1, 4, 6\} \subset \{1, 4, 6\}$

$$\{1, 4, 6\} \subseteq \{1, 4, 6\} \text{ or } \{1, 4, 6\} = \{1, 4, 6\}$$

(c) $\{a, b, c\}$ and $\{5, 10, 15\}$ are equal sets.

$$\{a, b, c\} \text{ and } \{5, 10, 15\} \text{ are equivalent sets.}$$

(d) $\{a, b, c, d, e\}$ has 25 subsets.

$$\{a, b, c, d, e\} \text{ has } 2^5 = 32 \text{ subsets.}$$

(e) $\emptyset \in \{6, 10, 20\}$

$$\emptyset \subset \{6, 10, 20\} \text{ or } \emptyset \subseteq \{6, 10, 20\}$$

(f) If $A = \{d : d \text{ is a day of the week ending in the letter "y"}\}$, then $n(A) = 1$.

$$\text{If } A = \{d : d \text{ is a day of the week ending in the letter "y"}\}, \text{ then } n(A) = 7.$$

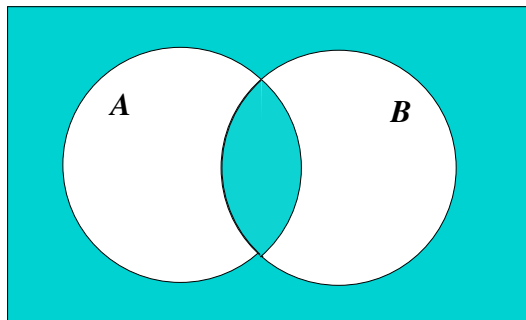
7. (6 points) List all two-element subsets of $\{A, G, J, R\}$.

$$\{A, G\}, \{A, J\}, \{A, R\}, \{G, J\}, \{G, R\}, \{J, R\}$$

8. (4 points) If $n(A) = 120$, $n(B) = 67$, and $n(A \cap B) = 14$, what is $n(A \cup B)$?

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 120 + 67 - 14 = 173$$

9. (4 points) Draw a Venn diagram with shading that illustrates the set $(A' \cap B') \cup (A \cap B)$.



10. (12 points) Let $\mathcal{U} = \{2, 3, 5, 7, 11, 13, 17\}$. Use $A = \{2, 3, 5, 7\}$, $B = \{5, 11, 17\}$, and $C = \{2, 11, 13, 17\}$ to determine the elements of the following sets. Be sure to show enough work to be eligible for partial credit.

(a) $(A - B) \cup C$

$$= \{2, 3, 7\} \cup \{2, 11, 13, 17\} = \{2, 3, 7, 11, 13, 17\}$$

(b) $(A' - C) \cup (B \cap A)$

$$= (\{11, 13, 17\} - \{2, 11, 13, 17\}) \cup \{5\} = \emptyset \cup \{5\} = \{5\}$$

(c) $\emptyset \cap (\mathcal{U} \cap (A - C) \cup (B' \cap C'))'$

$$= \emptyset$$

as anything intersected with the empty set is the empty set.