MATH 112 – FIRST EXAM SOLUTIONS

February 7, 2008

NAME: ______________________________

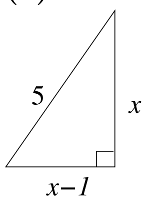
INSTRUCTIONS:

(1) SHOW ALL WORK.
(2) Do not begin until instructed to do so.
(3) You have 80 minutes to complete the exam.
(4) You may use a calculator but indicate any work prior to calculator use.
(5) When an **exact** answer is specified, a calculator approximation is not acceptable.
(6) Include units when appropriate.

PROBLEM	POINTS	SCORE
1	10	
2	6	
3	18	
4	9	
5	5	
6	6	
7	6	
8	12	
9	10	
10	10	
11	8	
TOTAL	100	

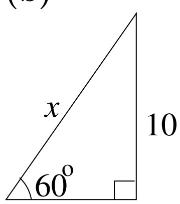
1. (10 points) Solve for x in each of the following triangles.

(a)



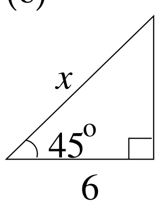
$$\begin{aligned} 5^2 = x^2 + (x-1)^2 &\Rightarrow 25 = x^2 + x^2 - 2x + 1 \\ &\Rightarrow 2x^2 - 2x - 24 = 0 \\ &\Rightarrow 2(x-4)(x+3) = 0 \\ &\Rightarrow x = 4 \text{ or } x = -3 \\ &\Rightarrow x = 4 \text{ (as } x > 0). \end{aligned}$$

(b)



Using the proportions in the $30° - 60° - 90°$ triangle, the side opposite $60°$ is $\sqrt{3}/2$ times the hypotenuse. Thus, $x = 20/\sqrt{3}$.

(c)



Using the proportions in the $45° - 45° - 90°$ triangle, the hypotenuse is $\sqrt{2}$ times the length of a side. Thus, $x = 6\sqrt{2}$.

2. (6 points) Name one positive and one negative angle that is coterminal with each of the given angles.

(a) $100°$ **(b)** $-10°$ **(c)** $200°$

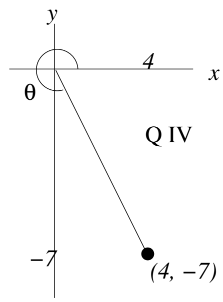
$100°$ is coterminal with $460°$ and $-260°$.

$-10°$ is coterminal with $350°$ and $-370°$.

$200°$ is coterminal with $560°$ and $-160°$.

3. An angle θ in standard position has the point $(4, -7)$ on its terminal side.

(a) (5 points) Draw θ and the point $(4, -7)$. Identify the quadrant to which θ belongs.



(b) (4 points) Find the distance from $(4, -7)$ to the origin.

We use the distance formula (or Pythagorean Theorem) to find:

$$r = \sqrt{4^2 + (-7)^2} = \sqrt{16 + 49} = \sqrt{65}.$$

(c) (9 points) Compute $\sin\theta$, $\cos\theta$, $\tan\theta$, $\sec\theta$, $\csc\theta$, and $\cot\theta$.

Using the point $(4, -7)$ and $r = \sqrt{65}$ from above, we have:

$$\begin{array}{ll} \sin\theta = -\dfrac{7}{\sqrt{65}} & \csc\theta = -\dfrac{\sqrt{65}}{7} \\ \cos\theta = \dfrac{4}{\sqrt{65}} & \sec\theta = \dfrac{\sqrt{65}}{4} \\ \tan\theta = -\dfrac{7}{4} & \cot\theta = -\dfrac{4}{7} \end{array}$$

4. (9 points) Suppose that angle θ belongs to quadrant II and that $\sin\theta = 3/\sqrt{10}$. Compute **exact** values for **(a)** $\cos\theta$, **(b)** $\tan\theta$, and **(c)** $\csc\theta$.

Using $\cos^2\theta + \sin^2\theta = 1$, we see that

$$\cos^2\theta = 1 - \frac{9}{10} = \frac{1}{10}.$$

Thus, $\cos\theta = \pm\dfrac{1}{\sqrt{10}}$. Since θ is in the second quadrant, the cosine value will be negative. Thus,

$$\cos\theta = -\frac{1}{\sqrt{10}} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3/\sqrt{10}}{-1/\sqrt{10}} = -3 \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{10}}{3}.$$

5. (5 points) Show that $\dfrac{\cos\theta}{\sec\theta} + \dfrac{\sin\theta}{\csc\theta} = 1$ by transforming the left side into the right side.

We convert everything into sines and cosines first and then simplify:

$$\frac{\cos\theta}{\sec\theta} + \frac{\sin\theta}{\csc\theta} = \frac{\cos\theta}{1/\cos\theta} + \frac{\sin\theta}{1/\sin\theta} = \cos^2\theta + \sin^2\theta = 1.$$

6. (6 points) Write the **exact** values for each of the following:

(a) $\sin 45^\circ = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ **(b)** $\cos 30^\circ = \dfrac{\sqrt{3}}{2}$ **(c)** $\tan 60^\circ = \dfrac{\sin 60^\circ}{\cos 60^\circ} = \dfrac{\sqrt{3}/2}{1/2} = \sqrt{3}$

7. (6 points) Compute the angle sums and then write the answer using decimal degrees rounded to the nearest hundredth of a degree.

(a) $(37^\circ 45') + (18^\circ 10')$

$$= 55^\circ 55' = 55^\circ + \left(\frac{55}{60}\right)^\circ = 55.92^\circ.$$

(b) $(65^\circ 37') + (10^\circ 52')$

$$= 75^\circ 89' = 76^\circ 29' = 76^\circ + \left(\frac{29}{60}\right)^\circ = 76.48^\circ.$$

8. If possible, find the angle θ between $0°$ and $90°$ for each statement below. If not possible, say what is wrong.

(a) (2 points) $\cos\theta = 0.546$

$$\theta = \cos^{-1}(0.546) = 56.9°.$$

(b) (2 points) $\sin\theta = 1.23$

$$\theta = \sin^{-1}(1.23) \text{ is undefined because } \sin\theta \text{ can never be larger than 1.}$$

(c) (4 points) $\csc\theta = 1.23$

$$\sin\theta = \frac{1}{1.23} = 0.813$$

and so

$$\theta = \sin^{-1}(0.813) = 54.4°.$$

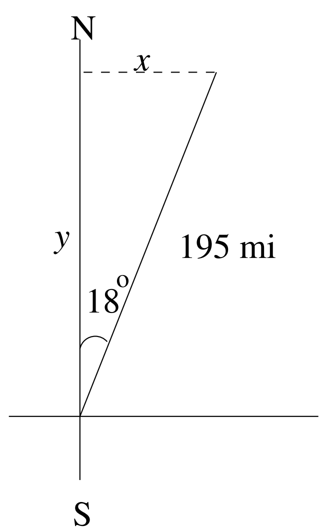
(d) (4 points) $\cot\theta = 4.63$

$$\tan\theta = \frac{1}{4.63} = 0.216$$

and so

$$\theta = \tan^{-1}(0.216) = 12.2°.$$

9. (10 points) A plane travels 195 miles at a bearing of N18°E. Find the total distance traveled north and total distance traveled east. Sketch a picture.



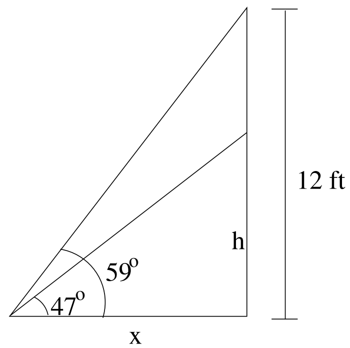
We need to find the values of x and y, which are the horizontal and vertical components of the vector representing the path of the airplane. Because of the position of the angle,

$$\sin 18° = \frac{x}{195} \text{ and } \cos 18° = \frac{y}{195}.$$

Solving for x and y, we find:

$x = 195\cos 18° = 60.3$mi and $y = 195\sin 18° = 186$mi.

10. (10 points) The angle of elevation from the floor to the top of a door is 47° while the angle of elevation to the ceiling is 59°. If the ceiling is 12 feet high, then what is the height h of the door? (If you need extra variables, label them in the picture.)



Label the distance from the floor to the bottom of the door as x. Then $\tan 59° = \frac{x}{12}$. Solving for x, we have

$$x = \frac{12}{\tan 59°} = 7.2 \text{ ft.}$$

Now, notice that $\tan 47° = \frac{h}{x}$ so that

$$h = x \tan 47° = 7.7 \text{ ft.}$$

11. (2 points each) Decide whether each statement is true or false and circle the correct answer.

(a) A triangle is called acute if all angles are acute. $\boxed{\textbf{TRUE}}$ **FALSE**

(b) A triangle is called obtuse if all angles are obtuse. **TRUE** $\boxed{\textbf{FALSE}}$

(c) If angles A and B are supplementary and $A = 35°$, then $B = 55°$. **TRUE** $\boxed{\textbf{FALSE}}$

(d) If $A = 35°$ and $B = 55°$, then $\csc A = \sec B$. $\boxed{\textbf{TRUE}}$ **FALSE**