Math 140 – Calculus I.
Extra Optimization Problems Answers

1. The sum of the squares is minimized when $x = 24$ and $y = 24$.

2. The cost is minimized when 40 computers are ordered.

3. When the perimeter is L, the area is maximized when length and width are both $\frac{L}{4}$ m.

4. The area of the rectangle is maximized when the length is 100 m and the width is $\frac{200}{\pi}$ m.
The area of the entire region is maximized when the length is 0 m and the width is $\frac{400}{\pi}$ m, i.e., when the track is just a circle.

5. The total area of the square and the circle are minimized when the square has perimeter of $\frac{8}{4+\pi}$ m and the circle has perimeter of $\frac{2\pi}{4+\pi}$ m.
To find the maximized total area, check the endpoints too, i.e., what is the area when all of the wire is used on the square and what is the area when all the wire is used on the circle? The maximum occurs when all of the wire is used on the circle. (Notice that both cases here result in total area larger than the minimum area above.)

6. The surface area is maximized when the base is $\frac{1}{2}$ meter by $\frac{1}{2}$ meter and the height is $\frac{1}{2}$ meter.

7. To minimize the surface area, the diameter should be $2\left(\frac{250}{\pi}\right)^{1/3}$ cm while the height should be $5\left(\frac{16}{\pi}\right)^{1/3}$ cm.
When the top and bottom cost twice as much as the sides, then count them double for the surface area formula. This cost is minimized when the diameter is $2\left(\frac{125}{\pi}\right)^{1/3}$ cm and the height is $\frac{20}{\pi^{1/3}}$ cm.

8. To find the minimum distance, it is easier to minimize the square of the distance. The closest point on $y = \sqrt{x}$ to $(1, 0)$ is $\left(\frac{1}{2}, \sqrt{\frac{1}{2}}\right)$.

9. The special part of this problem is recognizing that the total harvest H is a piecewise-defined function. When the total number of trees is $n \leq 60$, the total harvest is $120n$. When there are $n > 60$ trees, the total harvest is $n(120 - 2(n - 60))$. To maximize H, take the derivative of each piece, looking for critical points for the $n \leq 60$ piece and for the $n > 60$ piece. The common point, $n = 60$, should always be considered a critical point for piecewise functions.
It turns out that $n = 60$ trees is the optimal number of trees to maximize the total weight of fruit H.

10. The time for the 100 mile trip is $\frac{100}{v}$ hours. The cost of this trip is $C = \frac{2500}{v} + v$ dollars. This cost is minimized when the velocity v is a constant 50 mph.